The direction cosine matrix on p. 424 of Ref. 1 allows us to write

$$\hat{n} = \hat{n}_3 = S_i S_\theta \hat{l}_1 + S_i C_\theta \hat{l}_2 + C_i \hat{l}_3 \tag{22}$$

where S_i is shorthand for sin i, C_i is short for cos i, etc. Since $\hat{r} = \hat{l}_1$, we also have

$$\hat{r} \cdot \hat{n} = S_i S_\theta \tag{23}$$

Unit vectors $\hat{b}_1$, $\hat{b}_2$, and $\hat{b}_3$ are fixed in B, are assumed to be parallel to central principal axes of inertia of B, and are further assumed to be parallel to $\hat{l}_1$, $\hat{l}_2$, and $\hat{l}_3$. Thus, we can express $\underline{I}$ as

$$\begin{aligned}\underline{I} &= I_{11}\hat{b}_1\hat{b}_1 + I_{22}\hat{b}_2\hat{b}_2 + I_{33}\hat{b}_3\hat{b}_3 \\ &= I_{11}\hat{l}_1\hat{l}_1 + I_{22}\hat{l}_2\hat{l}_2 + I_{33}\hat{l}_3\hat{l}_3\end{aligned} \tag{24}$$

From Eqs. (22) and (24) we can see that

$$\underline{I} \cdot \hat{n} = I_{11} S_i S_\theta \hat{l}_1 + I_{22} S_i C_\theta \hat{l}_2 + I_{33} C_i \hat{l}_3 \tag{25}$$

and

$$\begin{aligned}\hat{n} \times \underline{I} \cdot \hat{n} = {} & (I_{33} - I_{22}) S_i C_i C_\theta \hat{l}_1 + (I_{11} - I_{33}) S_i C_i S_\theta \hat{l}_2 \\ & + (I_{22} - I_{11}) S_i^2 S_\theta C_\theta \hat{l}_3\end{aligned} \tag{26}$$

similarly

$$\hat{r} \times \underline{I} \cdot \hat{n} = 0\hat{l}_1 - I_{33} C_i \hat{l}_2 + I_{22} S_i C_\theta \hat{l}_3 \tag{27}$$

$$\hat{r} \times \underline{I} \cdot \hat{r} = \mathbf{0} \tag{28}$$

$$\hat{n} \times \underline{I} \cdot \hat{r} = 0\hat{l}_1 + I_{11}(C_i \hat{l}_2 - S_i C_\theta \hat{l}_3) \tag{29}$$

After substitution of Eqs. (23) and (26–29) into Eq. (1), we find that the body-basis measure numbers of $\boldsymbol{M}$ are

$$\boldsymbol{M} \cdot \hat{b}_1 = \boldsymbol{M} \cdot \hat{l}_1 = \frac{3\mu_\oplus J_2 R_\oplus^2}{R^5}(I_{22} - I_{33}) S_i C_i C_\theta \tag{30}$$

$$\boldsymbol{M} \cdot \hat{b}_2 = \boldsymbol{M} \cdot \hat{l}_2 = \frac{12\mu_\oplus J_2 R_\oplus^2}{R^5}(I_{11} - I_{33}) S_i C_i S_\theta \tag{31}$$

$$\boldsymbol{M} \cdot \hat{b}_3 = \boldsymbol{M} \cdot \hat{l}_3 = \frac{12\mu_\oplus J_2 R_\oplus^2}{R^5}(I_{22} - I_{11}) S_i^2 S_\theta C_\theta \tag{32}$$

In order to obtain an appreciation for numerical values of these measure numbers, we will consider central principal moments of inertia that are commensurate with a vehicle the size of a space station, and a radius of a conceivable circular space station orbit. The following values of astronomical constants for Earth, central principal moments of inertia, and orbital parameters are used:

$\mu_\oplus = 3.986 \times 10^5$ km^3/s^2
$J_2 = 1.08 \times 10^{-3}$
$R_\oplus = 6378$ km
$I_{11} = 1.355 \times 10^8$ kg-m^2
$I_{22} = I_{11}$
$I_{33} = \frac{1}{10} I_{11}$
$i = 28.5$ deg
$R = 6700$ km

We find that the measure numbers of the gravitational moment exerted about the mass center of our vehicle would vary with argument of latitude in the following manner:

$\boldsymbol{M} \cdot \hat{b}_1 = (0.1991$ N-m$)C_\theta$
$\boldsymbol{M} \cdot \hat{b}_2 = (0.7964$ N-m$)S_\theta$
$\boldsymbol{M} \cdot \hat{b}_3 = 0$

These numerical values become significant when one considers that moments of aerodynamic forces about the space station mass center are predicted to be in the neighborhood of 2 N-m, and the first term in Eq. (1) can produce values near 3 N-m for orientations of B in L other than the one considered in this example.

Conclusions

A method for obtaining vector-dyadic expressions for the gravitational moment about a body's mass center has been demonstrated. The demonstration has been accomplished by deriving an expression for the gravitational moment exerted by an oblate spheroid. The contribution of Earth oblateness to the gravitational moment exerted on a body has been evaluated numerically in a greatly simplified example. This contribution is significant in comparison with other external moments, such as the one produced by aerodynamic forces.

References

[1]Kane, T. R., Likins, P. W., and Levinson, D. A., *Spacecraft Dynamics,* McGraw-Hill, New York, 1983, pp. 112–157.

[2]Glandorf, D. R., "Gravity Gradient Torque for an Arbitrary Potential Function," *Journal of Guidance, Control, and Dynamics,* Vol. 9, Jan.–Feb. 1986, pp. 122–124.

[3]Wilcox, J. C., "Comment on 'Gravity Gradient Torque for an Arbitrary Potential Function'," *Journal of Guidance, Control, and Dynamics,* Vol. 10, March–April 1987, p. 224.

[4]Roberson, R. E., "Gravitational Torque on a Satellite Vehicle," *Journal of the Franklin Institute,* Vol. 265, Jan. 1958, pp. 13–22.

Tracking Accuracy for LEOSAT-GEOSAT Laser Links

Ramani Seshamani,* D. V. B. Rao,* T. K. Alex,† and Y. K. Jain‡
ISRO Satellite Centre, Bangalore, India

Introduction

THE importance of laser-based intersatellite communication links (LASERCOM) has emerged recently. Keeping in mind its advantages, several workers[1–3] have reported on the antenna diameters and the pointing, acquisition, and tracking (PAT) accuracy requirements of laser intersatellite links. Particularly with regard to laser-diode-based intersatellite links, Boutemy et al.[2] have assumed a pointing accuracy of 1 μrad for the low-Earth-orbit to geostationary-orbit link. Popescu et al.[3] have postulated a beam divergence of 3 μrad.

As the actual values of the beam divergence and the tracking accuracy requirements for various antenna diameters and signal-to-noise ratios (S/N) have to be known for design of the LASERCOM, the values are calculated using the Gaussian beam characteristics of the laser.

Methodology

Since the laser beam has a Gaussian intensity profile, the following relations[4] hold:

$$w^2(z) = w_o^2[1 + (\lambda z / \pi w_o^2)^2] \tag{1}$$

$$\theta = \lambda / \pi w_o \tag{2}$$

Received March 25, 1988; revision received May 6, 1988.

*Engineer, Sensor Systems Division.
†Head, Sensor Systems Division.
‡Deputy Head, Sensor Systems Division.

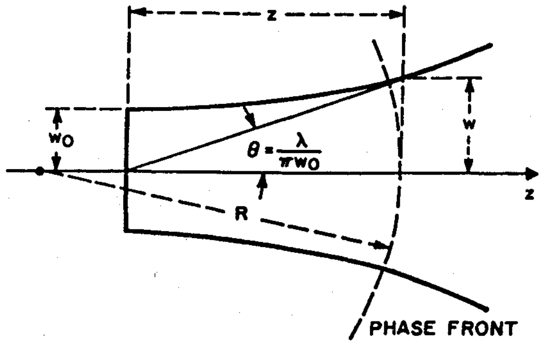

Fig. 1 Gaussian characteristics of a laser beam.

where (Fig. 1)

w = distance at which the amplitude is $1/e$ times that on the axis
w_o = radius of the beam waist
θ = semiangle between the asymptotes to the beam contour, and the axis, equal to the far-field diffraction angle of the fundamental mode
λ = wavelength

Assuming afocal optics at the transmitter and the receiver, the beam waist is located at the primary and w_o = radius of the primary.

Based on the minimum power required by the detector[5] (N_D), for a bit error rate (BER) of 10^{-8} and a data rate of 200 Mbps, S/N is calculated as follows:

$$S/N = PR^2 L_T L_R / (w^2 N_D) \tag{3}$$

$$\beta = 2\theta \tag{4}$$

where

P = laser peak pulse power
R = GEOSAT receiver optics entrance aperture radius
L_T = optical loss at transmitter optics
L_R = optical loss at receiver optics
β = beam divergence angle

We assume, further, that the data rate is 200 Mbps as proposed by Boutemy et al.,[2] the transmitted pulse power is 5 W at 820 nm, and the detector is a PIN photodiode.

At the LEOSAT end the transmitter has to swing ± 115 deg, hence there is a constraint on the antenna (mirror) weight and, hence, diameter, which is the predominant constraint. The total swing at the GEOSAT is ±10 deg; its antenna weight and diameter is not as critical. Furthermore, the GEOSAT has a wide-angle laser beacon which is acquired by the LEOSAT receiver as the first step in the acquisition procedure. Hence, the GEOSAT receiver mirror diameter can be taken as 250 mm. The S/N ratios for various β are calculated using the preceding equations for an intersatellite distance (z) of 45,000 km^2 and various transmitter primary mirror radii (w_o).

Results

From Eqs. (1–4) and the following values for the parameters: $P = 5$ W, $R = 125$ mm, $N_D = 2 \times 10^{-6}$ W, $\lambda = 0.00082 \times 10^{-3}$ m, and $L_T = L_R = 0.5$, the S/N ratio is calculated for various LEOSAT optics sizes and the results are tabulated in Table 1.

The following parameters are assumed: Bit error rate = 10^{-8} and data rate = 200 Mbps.

Table 1 gives the values of the radius (w) of the beam at z = 45,000 km (maximum LEOSAT-GEOSAT intersatellite distance), and S/N and β for various w_0.

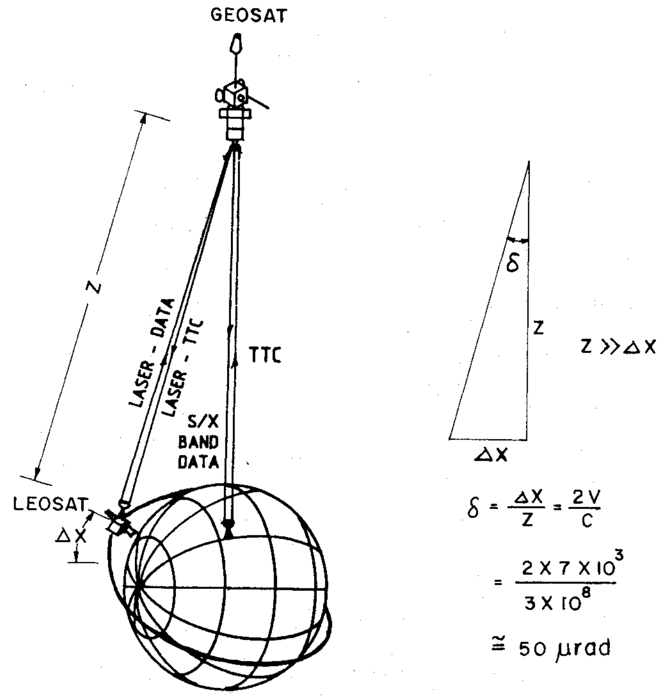

Fig. 2 Laser LEOSAT-GEOSAT communication link-point-ahead.

Table 1 Signal-to-noise ratios and beam divergences for various primary mirror radii

w_o (× 10_3), m	w, m	S/N	β, μrad
10	1174.56	0.0071	52.2
20	587.28	0.028	26.1
50	234.91	0.12	10.4
100	117.46	0.71	5.2
125	93.96	1.1	4.2

Table 1 shows that for an S/N > 1, a 125 mm radius primary optics is required at the LEOSAT, and the beam divergence must be less than 4.2 μrad. The tracking accuracy has to be within a fraction[6] of this, for example, 1 μrad, to make optimum use of the narrow beam divergence obtainable, and to keep pointing loss small.

Discussion

It is seen that, in the practical situation, tracking accuracy may pose a constraint on antenna diameter. By cooperative fine tracking,[7] a tracking accuracy better than 1 μrad can be achieved.

Assuming 0.1 deg (1.75 mrad) stability for a typical spacecraft, an interactive mode of closed-loop tracking would have to be implemented to achieve the required tracking accuracy.

Further, because of the small beam divergence, the point-ahead angle caused by the finite velocity of light would have to be taken into account. Figure 2 shows a schematic of the LEOSAT-GEOSAT communication link for laser data and telemetry/telecommand (TTC) and the point-ahead angle[8] (δ) geometry. The point-ahead angle is given by the following equation:

$$\delta = \frac{\Delta X}{z} = \frac{2V}{c} \tag{5}$$

where

ΔX = the distance travelled by the LEOSAT in the time interval (τ) required for the wavefront to traverse the intersatellite distance (z) ($z \gg X$)
V = velocity of the LEOSAT
c = velocity of light = 3×10^8 ms^{-1}

Based on a value of $V = 7$ km/s, the value of the point-ahead angle works out as $\delta \simeq 50\ \mu$rad.

Further, a duplexing method, either by polarization- or wavelength-duplexing,[2] would have to be implemented to avoid crosstalk.

Conclusions

This Note has shown that tracking accuracy of 1 μrad has to be achieved for LEOSAT-GEOSAT laser communication links. This requires a high degree of alignment between the transmitter and receiver and point-ahead capability.

Fine-pointing mirrors would have to be used with an angular range of about 1 deg. The pointing and acquisition procedure would have to be effected as follows: Initially, the two optical systems/telescopes would have to be pointed toward each other with an attitude accuracy smaller than the position uncertainty. Acquisition would start with the GEOSAT telescope's beamwidth deliberately widened to illuminate the LEOSAT in spite of the position uncertainty. A spatial scan operation by the narrow beam of the LEOSAT and, subsequently, by the GEOSAT would have to be carried out before acquisition is completed and switching from the acquisition mode to the tracking mode can take place. For tracking, a monopulse system with quadrant photodetectors or a conical scan system with single-element detectors can be used.

Acknowledgment

We would like to thank the Director of the ISRO Satellite Centre for his encouragement of the work.

References

[1]Lopez, J. M. and Yong, K., "Laser Beam Pointing Control Acquisition and Tracking Systems," *Laser Satellite Communications*, edited by M. Katzman, Prentice-Hall, Englewood Cliffs, NJ, 1987, pp. 190–213.

[2]Boutemy, J. C., Cousin, B., Frecon, L., and Sein, E. "Liaisons Optiques Intersatellites dans le cadre de la mission STAR," European Space Agency, Paris, SP-202, 1984, pp. 91–97.

[3]Popescu, A. F., et al. "Laboratory Model of a Bidirectional Diode Laser Data Link with Acquisition and Tracking Capability," *Society of Photo-Optical Instrumentation Engineers, Vol. 810, Optical Systems for Space Applications*, Bellingham, WA, 1987, pp. 239–244.

[4]Kogelnik, H., "Propagation of Laser Beams," *Applied Optics and Optical Engineering* Vol. 7, edited by R. R. Shannon and J. C. Wyant, Academic, New York, 1979, pp. 156–190.

[5]Iizuka, K., *Engineering Optics*, Springer-Verlag, Berlin, 1985.

[6]Leeb, W. R., "Prospects of Laser Communications in Space," European Space Agency, Paris, SP-202, May 1984, pp. 3–13.

[7]Ross, M., Wolf, J. D., Linford, R. M. F., and Abernathy, J. L., "Space Laser Communications Systems for the Eighties," *Proceedings of the 8th Communications Satellite System Conference*, AIAA, New York, 1980.

[8]Englisch, W., "Study on Intersatellite Laser Communication Links, Final Report," European Space Agency, Paris, ESTEC Contract 3555/78, July 1979.